## EECE 450 - Engineering Economics - Formula Sheet

Cost Indexes:
$\underline{\text { Cost at time } \mathrm{A}}=\underline{\text { Index value at time } \mathrm{A}}$
$\overline{\text { Cost at time B }}=\overline{\text { Index value at time B }}$
Power sizing:
$\frac{\text { Cost of asset A }}{\text { Cost of asset B }}=\left[\frac{\text { Size (capacity) of asset A }}{\text { Size (capacity) of asset B }}\right]^{x}$
$x=$ power - sizing exponent
Learning Curve:
$T_{N}=T_{\text {initial }} \times N^{b}$
$b=\frac{\log (\text { learning curve rate })}{\log 2}$
$T_{N}=$ time to make $N$ th unit
$T_{\text {initial }}=$ time to make first unit
$N=$ number of finished units
$b=$ learning curve exponent

## Simple Interest:

Interest earned on amount $P: I=$ Pin
Maturity value: $F=P(1+i n)$
$i=$ interest rate per time period
$n=$ number of time periods

## Compound Interest:

$F=P(1+i)^{n}$
$F=$ future value
$P=$ present value
$i=$ periodic interest rate
$n=$ number of periods
Ordinary Simple Annuity:
$P=A\left[\frac{1-(1+i)^{-n}}{i}\right]$
$F=A\left[\frac{(1+i)^{n}-1}{i}\right]$
$A=$ periodic payment (end of period)
$P, F, i, n$ as above for compound interest
Ordinary Arithmetic Gradient Annuity:
$A_{e q}=G\left[\frac{1}{i}-\frac{n}{(1+i)^{n}-1}\right]$
$P=G\left[\frac{(1+i)^{n}-i n-1}{i^{2}(1+i)^{n}}\right]$
$A_{e q}=$ equivalent periodic payment
$G=$ gradient amount (periodic increment)
$P, i, n$ as above for compound interest

Ordinary Geometric Gradient Annuity:
$P=A_{1}\left[\frac{1-(1+g)^{n}(1+i)^{-n}}{i-g}\right] ; i \neq g$
$P=\frac{n A_{1}}{(1+i)} ; i=g$
$F=A_{1}\left[\frac{(1+i)^{n}-(1+g)^{n}}{i-g}\right] ; i \neq g$
$F=n A_{1}(1+i)^{n-1} ; i=g$
$A_{1}=$ payment in first period (end)
$g=$ periodic rate of growth
$P, F, i, n$ as above for compound interest
Simple Annuity Due:
$P=A\left[\frac{1-(1+i)^{-n}}{i}\right](1+i)$
$F=A\left[\frac{(1+i)^{n}-1}{i}\right](1+i)$
$A=$ cash amount (beginning of period)
$P, F, i, n$ as above for compound interest
Nominal, Periodic, Effective Interest Rates:
$i=\frac{r}{m}$
$\left(1+i_{\text {eff }}\right)=\left(1+\frac{r}{m}\right)^{m}$
$r=$ nominal interest rate per year
$m=$ number of compounding periods per year
$i_{\text {eff }}=$ effective interest rate (compounded annually)
$i=$ periodic interest rate

## Equivalent Interest Rates:

$\left(1+i_{p}\right)^{p}=\left(1+i_{c}\right)^{c}$
$i_{p}=$ interest rate for payment period
$p=$ number of payment periods per year
$i_{c}=$ interest rate for compounding period
$c=$ number of compounding periods per year
Ordinary General Annuity:
$P=A\left[\frac{1-\left(1+i_{p}\right)^{-n}}{i_{p}}\right]$
$F=A\left[\frac{\left(1+i_{p}\right)^{n}-1}{i_{p}}\right]$
$i_{p}=$ interest rate for payment period
$n=$ number of payment periods
$P, F, A$ as above for annuities

## Perpetual Annuities:

Ordinary : $P=\frac{A}{i}$
Due : $P=\frac{A}{i}(1+i)=\frac{A}{i}+A$
Geometric Growth : $P=\frac{A}{i-g} ; i>g$

## $P, A, i, g$ as above for annuities

## Investment Criteria:

$\mathrm{NPV}=C F_{0}+\frac{C F_{1}}{(1+r)^{1}}+\frac{C F_{2}}{(1+r)^{2}}+\ldots+\frac{C F_{n}}{(1+r)^{n}}$
$\mathrm{NPV}=$ net present value
$\mathrm{NFV}=C F_{0}(1+r)^{n}+C F_{1}(1+r)^{n-1}+\ldots+C F_{n}$
$\mathrm{NFV}=$ net future value
$\mathrm{EACF}=$ equivalent annual cash flow $=\frac{\mathrm{NPV}}{\left[\frac{1-(1+r)^{-n}}{r}\right]}$
$C F_{j}=$ cash flow at time $j$
$n=$ lifetime of investment
$r=$ MARR $=$ minimum acceptable rate of return
$0=C F_{0}+\frac{C F_{1}}{(1+i)^{1}}+\frac{C F_{2}}{(1+i)^{2}}+\ldots+\frac{C F_{n}}{(1+i)^{n}}$
$i=\mathrm{IRR}=$ internal rate of return
$\operatorname{PV}\left(\right.$ neg CFs, $\left.\mathrm{e}_{\text {fin }}\right) \times\left(1+i^{\prime}\right)^{n}=\mathrm{FV}\left(\operatorname{posCFs}, \mathrm{e}_{\text {inv }}\right)$
$i^{\prime}=$ MIRR $=$ modified internal rate of return
$\mathrm{e}_{\text {fin }}=$ financing rate of return
$e_{\text {inv }}=$ reinvestment rate of return
Benefit - cost ratio, $\mathrm{BCR}=\frac{\mathrm{PV} \text { (positive cash flows) }}{\mathrm{PV}(\text { negative cash flows })}$
Probability:
$\mathrm{E}(X)=$ Weighted average $=\frac{w_{1} S_{1}+\cdots+w_{k} S_{k}}{w_{1}+\cdots+w_{k}}$
$w_{i}=$ weight for Scenario $i$
$S_{i}=$ value of $X$ for Scenario $i$
$\mathrm{E}(X)=\mu_{X}=$ expected value of $X=\sum_{\text {all } j} \mathrm{P}\left(x_{j}\right) x_{j}$
$\operatorname{Var}(X)=$ variance of $X=\sum_{\text {all } j} \mathrm{P}\left(x_{j}\right)\left(x_{j}-\mu_{X}\right)^{2}$
$\mathrm{P}\left(x_{j}\right)=\operatorname{Probability}\left(X=x_{j}\right)$

## Depreciation:

$B=$ initial (purchase) value or cost basis
$S=$ estimated salvage value after depreciable life
$\mathrm{d}_{\mathrm{t}}=$ depreciation charge in year t
$\mathrm{N}=$ number of years in depreciable life
Book value at end of period $\mathrm{t}: \mathrm{BV}_{\mathrm{t}}=\mathrm{B}-\sum_{i=1}^{t} d_{i}$
Straight-Line (SL):
Annual charge: $d_{t}=(B-S) / N$
Book value at end of period $t: B V_{t}=B-t \times d$

Sum-of-Years'-Digits (SOYD):
SOYD $=\mathrm{N}(\mathrm{N}+1) / 2$
Annual charge: $d_{t}=(B-S)(N-t+1) /$ SOYD
Declining balance (DB):
$D=$ proportion of start of period $B V$ that is depreciated
Annual charge: $d_{n}=B D(1-D)^{n-1}$
Book value at end of period $n$ : $B V_{n}=B(1-D)^{n}$
Capital Cost Allowance (CCA):

## $\mathrm{d}=\mathrm{CCA}$ rate

$\mathrm{UCC}_{\mathrm{n}}=$ Undepreciated capital cost at end of period $n$
Annual charge: $\mathrm{CCA}_{1}=\mathrm{B}(\mathrm{d} / 2)$ for $\mathrm{n}=1$;
$\mathrm{CCA}_{\mathrm{n}}=\mathrm{Bd}(1-\mathrm{d} / 2)(1-\mathrm{d})^{\mathrm{n}-2}$ for $\mathrm{n} \geq 2$
UCC at end of period $\mathrm{n}: \mathrm{UCC}_{\mathrm{n}}=\mathrm{B}(1-\mathrm{d} / 2)(1-\mathrm{d})^{\mathrm{n}-1}$
$\operatorname{PV}(C C A$ tax shields gained $)=\left[\frac{B d T_{C}}{i+d}\right]\left[\frac{1+i / 2}{1+i}\right]$
$\operatorname{PV}($ CCA tax shields lost $)=\left[\frac{S d T_{C}}{i+d}\right]\left[\frac{1}{(1+i)^{N}}\right]$
$T_{C}=$ firm's tax rate $; i=$ discount rate

## Investment Project Cash Flows:

Taxable income $=\mathrm{OR}-\mathrm{OC}-\mathrm{CCA}-\mathrm{I}$
Net profit $=$ taxable income $\times(1-\mathrm{T})$
Before-tax cash flow (BTCF) $=\mathrm{I}+\mathrm{CCA}+$ taxable income
After-tax cash flow $($ ATCF $)=$ Net profit + CCA + I

$$
\begin{aligned}
& =(\text { Taxable income }) \times(1-\mathrm{T})+\mathrm{CCA}+\mathrm{I} \\
& =(\mathrm{BTCF}-\mathrm{I}-\mathrm{CCA})(1-\mathrm{T})+\mathrm{CCA}+\mathrm{I} \\
& =(\mathrm{OR}-\mathrm{OC})(1-\mathrm{T})+\mathrm{I}(\mathrm{~T})+\mathrm{CCA}(\mathrm{~T})
\end{aligned}
$$

Net cash flow from operations

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\begin{aligned}
& =\text { ATCF }-\mathrm{I}-\text { DIV } \\
& =(\mathrm{OR}-\mathrm{OC})(1-\mathrm{T})+\mathrm{I}(\mathrm{~T})+\mathrm{CCA}(\mathrm{~T})-\mathrm{I}-\mathrm{DIV} \\
& =(\mathrm{OR}-\mathrm{OC}-\mathrm{I})(1-\mathrm{T})+\mathrm{CCA}(\mathrm{~T})-\text { DIV } \\
& =\text { Net profit }+\mathrm{CCA}-\text { DIV }
\end{aligned}
$$

$\mathrm{OR}=$ operating revenue; $\mathrm{OC}=$ operating cost
$\mathrm{I}=$ interest expense; DIV = dividends; $\mathrm{T}=$ tax rate
Net cash flow $=$ Net cash flow from operations

+ New equity issued + New debt issued
+ Proceeds from asset disposal - Repurchase of equity
- Repayment of debt (principal) - Purchase of assets

Net capital investment $=B\left[1-\frac{d T_{C}}{i+d} \frac{1+i / 2}{1+i}\right]$
Net salvage value $=S\left[1-\frac{d T_{C}}{i+d}\right]\left[\frac{1}{(1+i)^{N}}\right]$
Inflation:
$(1+i)=(1+i)(1+f)$
$i=i^{\prime}+f+(i)(f)$
$i=$ market interest rate; $i=$ real interest rate
$f=$ inflation rate
Weighted Average Cost of Capital (WACC):
$\mathrm{WACC}=\frac{D}{V} \times\left(1-T_{C}\right) i_{d}+\frac{E}{V} \times i_{e}$
$V=D+E$
$D=$ market value of debt; $E=$ market value of equity
$V=$ market value of firm
$i_{d}=$ cost of (rate of return on) debt
after-tax cost of debt: $i_{d t}=i_{d}(1-\mathrm{T})$
$i_{e}=$ cost of equity

